"When Nothing Remains...:" A Discussion of the Evolution of a Conceptual Definition of Zero

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Reckoning with a functional definition of zero, a number, and a conceptual definition of "nothingness," brought mathematics dangerously close to theology for over a thousand years. This tension seems particularly ironic when viewed through the lens of the 21st century, a time where mathematics is perceived to be pure truth, void of mysticism and religious doctrine. However, every culture is shaped by its common beliefs and every era of history reflects those tenets. An explanation of why the ancient Greeks missed the functional need for zero and why Hindu mathematicians recognized this need and accepted its conceptual implications may be found within the context of the different ideologies of the cultures.

For the western world, the confusion called zero began about 1,200 years ago. Sometime around 800 AD, a mathematician and court astronomer named Mohammed ibn Musa al-Khowarizmi was working for Caliph al-Ma'mun in the House of Wisdom in Baghdad. Al-Khowarizmi produced two brief works which introduced to the west the Hindu concept of numerals and an algebraic approach to mathematics.[1] His discourse on arithmetic is credited as the first published work to document the Hindu decimal system, which used symbols for the first nine digits. In it, he added this fateful idea:

> *When nothing remains [in subtraction], put down a small circle so that the place be not empty but the circle must occupy it.*[2]

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[1]David Burton, The History of Mathematics: An Introduction (Boston: WCB/ McGraw-Hill, 1990), p.227.

[2] Burton, p.227.

With this single line, the shape of mathematics was to be forever changed, although the western world would wrestle with the conceptual implications of zero for hundreds of years. Not until the end of the 19th century would number theory, the "core of mathematics," be organized around the Peano postulates which define the fundamental laws of positive numbers and begin with the definitive statement: "0 is a number."[3]

Although al-Khowarizmi is credited with introducing a functional definition of zero to the west, the concept was by no means his original idea. Hindu mathematicians working between 200 and 700 AD are credited with introducing the symbol for zero and laying a foundation for the conceptual definition of its function. In fact, the concept of negative numbers is introduced in a few Hindu tablets, serving the very practical purpose of listing outstanding debts.[4] A symbol denoting a vacant space within the sexagesimal representation of numerals is found in Babylonian tablets produced between 400 and 300 BC.[5] There are also symbols denoting a vacant space in the sexagesimal fractions in Greek papyri dating from 300 to 100 BC.[6] Continents removed, two symbols denoting an empty space (placeholders) are found in Mayan hieroglyphics dating from the fourth century AD.[7]

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[3] Ranjit Ramchandra Desai, "Logic," The New Encyclopedia Britannica, 15th ed. (1998), Vol. 23, p. 275.

[4] Louis Charles Karpinski, The History of Arithmetic (New York: Russell & Russell, 1965), p. 41.

[5] Raymond L. Wilder, Evolution of Mathematical Concepts: An Elementary Study (New York: John Wiley & Sons, Inc., 1968), p.52.

[6] Morris Kline, Mathematical Thought from Ancient to Modern Times (New York: Oxford University Press, 1972), p. 184.

[7] John McLeish, The Story of Numbers: How Mathematics Has Shaped Civilizations (New York: Fawcett Columbine, 1991), p. 131.

While ancient Greek mathematicians may have acknowledged the need to denote a vacant space within the representation of a number, this awareness did not lead to comprehension of the concept of the number zero. Western culture has been historically and instinctively enamored with the art, science, and philosophy of ancient Greece. In A Mathematician's Apology, G.H. Hardy writes: "The Greeks were the first mathematicians who are still 'real' to us to-day. Oriental mathematicians may be an interesting curiosity, but Greek mathematics is the real thing."[8] Hardy's breezy generalization is an acknowledgement of the fundamental structure of Greek mathematics, which divided the field into arithmetic (the study of "multitude," or discrete quantity) and geometry ("magnitude," or continuous quantity) and "considered both to have originated in practical activities."[9] The origins of Greek mathematics (arithmetic and geometry) are undoubtedly rooted in the mathematics of ancient Egypt, since the Greeks maintained commercial and military relations with Egypt from the 7th century BC.[10]

Hardy's affinity with the ancient Greek mathematicians is explained by their insistence on general statements that could be confirmed by proof; but Greek mathematicians were hindered by their cosmology. The Pythagoreans believed that "everything is number."[11] Physical shapes were interpreted mathematically, and the Pythagoreans believed it was the number associated with the shape that provided matter with form and structure. This belief is behind Aristotle's fundamental assertion:

> "the elements of number are the even and the odd, and that of these the latter is limited, and the former unlimited; and that the One proceeds from both of these (for it is both even and odd), and number from the One; and that the whole heaven, as has been said, is numbers."[12]

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[8] G. H. Hardy, A Mathematician's Apology (1940, rpt. Cambridge University Press, 1992), p. 81.

[9] Wilbur R. Knorr, "The History of Mathematics," The New Encyclopedia Britannica (1998), Vol. 23, p. 579.

[10] Knorr, p. 579.

[11] Don E. Marietta, Jr. Introduction to Ancient Philosophy (London: M.E. Sharpe, Inc., 1998), p. 15.

[12] "Testimonia on Pythagoras," by Aristotle as quoted in Drew Hyland, The Origins of Philosophy: Its Rise in Myth & the Pre-Socratics (New York: Capricorn Books, 1973), p. 141.

For a society whose belief structure rested on the idea that "the whole of heaven is numbers," it is, as Spengler asserts in his treatise *The Meaning of Numbers*, "understandable that even *negative* numbers, which to us offer no conceptual difficulty, were impossible to the Classical mathematic, let alone *zero as a number*, that refined creation of a wonderful abstractive power which, for the Indian soul that conceived it as a base for a positional numeration, was nothing more nor less than the key to the meaning of existence. *Negative magnitudes* have no existence."[13]

Distance, area, and volume were the concepts of the tangible, physical world. The equations to calculate these numbers were understood, defined, and proven by Greek mathematicians. To consider a fourth-degree equation, much less a negative number, would be absurd in the context of their philosophical definition of reality. To the ancient Greeks, that which was beyond the eye did not exist. In fact, there was no word in the language to define the concept of "space."[14]

The notion of nothingness was contemplated by the Eleatic philosophers who lived in the fifth century BC. The Eleatics believed that the appearance of change is an illusion.[15] A quote attributed to a prominent Eleatic, Melissus of Samos, suggests one of their fundamental difficulties with the concept of nothingness:

> "What has been has always been, and will always be. For if it had come into being, before its coming into being it would without fail have been nothing. If therefore it was nothing, it could not in any way have become something out of nothing.'"[16]

Parmenides, another of the Eleatics, reasoned that nothingness could not exist because nothingness could not be thought; if something could not be thought, it could not exist and there could be no

[13] Oswald Spengler, "Meaning of Numbers," in Vol. 4 of The World of Mathematics, ed. James R. Newman (New York: Simon & Schuster, 1956), p.2325.

[14] Spengler, p. 2340.

[15] Marietta, p. 22.

[16] Melissus, frg. B I, from Arnold Ehrhardt The Beginning: A Study of the Greek Philosophical Approach to the Concept of Creation (Manchester University Press, 1968), p. 7.

proof of the existence of nothingness.[17] This reality is perceived to be eternal and changeless, spawned from One. Translated into literal, numerical reality, the number one functioned as the creator of all numbers for the ancient Greeks. How could “nothing” have “one” as a successor?

A step towards the possibility of acknowledging and defining "nothingness" is found in the philosophy of the Atomists, who followed the Eleatics. The revolutionary idea in Atomist thought is the suggestion of “not-being.” The Atomists postulated that atoms (associated with “being”) and void (associated with “not-being”) were alike. Acceptance of the concept of zero would have to be based on reason, as opposed to physical experience. Atomist philosophy helped prepare western thought for this concept.

Given the ancient Greek's rigid view of physical reality, reflected in their development of plane geometry, it is not surprising that these innovative and curious mathematical minds missed the functional need for the number zero.

The functional need for zero, and a conceptual definition of “nothingness,” was not as troubling to the ancient Hindu astronomers, mathematicians, and philosophers. Considered within the context of the philosophical tradition, al-Khowarizmi's seemingly casual remark makes sense.

The ancient Hindus and ancient Greeks shared a common fundamental belief in an underlying moral order within the universe.[18] From the *Vedas*, the oldest scriptures of India (dating as far back as 500 BC[19]) come explanations of Hindu views on creation. The fundamental belief is that creation has no beginning and no end.[20] A discussion of Hindu ideology offers an argument strikingly similar to the Eleatics:

> "If creation had a beginning, then must the creator also have had a beginning, since until there is a creation there can be no creator; but to admit that the creator had a beginning would be to admit that God had a beginning, since God is not God until he creates -- and to

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[17]Marietta, p. 24.

[18] Timothy J. Lomperis Hindu Influence on Greek Philosophy (Calcutta: Minerva Associates, 1984), p. 15.

[19] Sources of Indian Tradition, Vol 1, ed. Wm Theodore de Bary (New York: Columbia University Press, 1958), p. 2.

[20] Swami Prabhavananda The Spiritual Heritage of India (Hollywood, CA: Vendanta Press, 1979), p. 27.

think of God as having a beginning would, to the Hindu, be a manifest absurdity."[21]

Where Hindu thought diverges from Greek thought is in the acceptance of the idea that "God" is eternal and beyond the cycling of birth and death. The tacit implication then is that "God" is beyond the cycles of the numbers of the physical world. In this manner, the spiritual teachings of the *Vedas* provided Hindu thinkers with the cultural permission to accept the concept of zero, as a number, without contradicting the concept of "nothingness."

This general idea was a subject contemplated by Ramanujan, arguably the greatest number theorist of our time. According to an account in the biography The Man Who Knew Infinity:

> "Ramanujan was fascinated by the quantity 2^n minus 1. That, a friend remembered him explaining, stood for 'the primordial God and several divinities. When *n* is zero, the expression denotes zero, there is nothing; when *n* is 1 the expression denotes unity, the Infinite God. When *n* is 2, the expression denotes Trinity; when *n* is 3, the expression denotes 7, the Saptha Rishis, and so on.'"[22]

Ramanujan also attempted to construct a theory of reality relating zero and infinity. "Zero, it seemed, represented Absolute Reality. Infinity was the myriad manifestations of the Reality. Their mathematical product was not one number, but all numbers, each of which corresponded to individual acts of creation."[23] Due to Ramanujan's brilliance and his decidedly different cultural traditions, these idiosyncrasies were over-looked by his British friends and colleagues who surely suppressed a desire to roll their eyes when he made statements like: "'An equation for me has no meaning unless it expresses a thought of God.'"

For the Hindu, finding an "expression of God" in an equation is a natural and expected occurrence. A philosophical tradition that recognizes "God" in all things, particularly in the secular art of mathematics, might have provided the permission and mental flexibility to see the need for both a symbol and a concept for representing and defining what results "when nothing remains."

The development of this symbol and definition, what we now know as zero, with its simple

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[21] Prabhavananda, p. 27.

[22] Robert Kanigel, The Man Who Knew Infinity: A Life of the Genius Ramanujan (New York: Washington Square Press, 1991), p. 66.

[23] Kanigel, p. 66.

and obvious properties, has been described as "one of the most important events in the history of mathematics."[24]

The word "zero" is derived from the Arabic *sifr* ("something empty"). In turn, *sifr* comes from the Hindu word *sunya*. While ancient Hindu mathematicians may have used *sunya* in a strictly notational sense, there is no escaping the theological implications of this definition. From ancient texts on Buddhism (for which the first written records date to 80 BC[25]) comes the doctrine of *Sunyata*, meaning "Emptiness" or "the Void." The following passage, from the *Multitude of Graceful Actions*, a lyrical account of the life of Buddha, provides some insight into the implications of meaning of *Sunyata*:

> "So the turning of all the components of becoming
> Arises from the interaction of one with another.
> In the unit the turning cannot be traced
>   Either at the beginning or end.
> ...
> The mystic knows the beginning and end
>   Of Consciousness, its production and passing away --
> He knows that it came from nowhere and returns to nowhere,
>   And is empty [of reality], like a conjuring trick.[26]

The modern word "cipher" is defined as:

> **1** the symbol 0, indicating a value of naught; zero **2** a person or thing of no importance or value; nonentity **3** a) a system of secret writing based on a key or symbols b) a message in such writing c) the key to such a system **4** an intricate weaving together of letters, as a monogram **5** an Arabic numeral - ***vi.*** **1**[Now Rare] to solve by arithmetic **2** to express in secret writing.[27]

In this 20th century dictionary definition is nearly a concise history of the integration of

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[24] David Eugene Smith & William Judson LeVeque, "Arithmetic," The New Encyclopedia Britannica (1998), Vol. 14, p. 75.

[25] Prabhavananda, p. 172.

[26] de Bary, p. 173-174.

[27] Webster's New World Dictionary, 3rd Ed. (Cleveland: Webster's New World, 1988), p. 254.

this concept from Arabic to western thought. Even for the Arabic mathematicians who accepted the concept, the idea of negative numbers was an uncomfortable implication -- except for denoting the amount of money one was owed. For example, while recognizing the existence of two solutions to a quadratic equation, these scholars listed only the positive ones. As David Burton writes, "The very idea of a negative root implies the acknowledgement of negative numbers as independent entities having the same mathematical status as positive ones."[28]

Even after European mathematics began to accept the functional definition of zero, there was a lingering temptation to equate zero the number with "nothingness." In the mid-to-late-17th century, Leibniz asserted that God had created the universe (1) out of nothing (0).[29] Two hundred years later, the debate would still be active. Perhaps this idea motivated Kronecker's assertion that "The natural numbers are the only numbers that assuredly exist. They are given to use by the Almighty. Everything else is the work of man."[30]

In modern mathematics, the conceptual and functional definitions of zero are taken without question. Zero is the "special" or "trivial" case. But echoing the implications found in the modern definition of the word "cipher," peculiarities abound if we scratch the surface of the properties of zero. For example, "symbols such as x^0 and 0! (factorial zero) have to be given stipulated definitions that are not derivable from any natural equation of 'zero' with 'no' or with 'nothing.'"[31] As Constance Reid points out in From Zero to Infinity, "Zero is the only number which can be divided by every other number, and the only number which can divide no other number."[32]

Perhaps there is more to be mined from the conceptual definition of zero, as Ramanujan suspected. It seems ironic that a concept which, by its modern mathematical definition, is "trivial" and "obvious" was so difficult for ancient and even medieval mathematicians to assimilate.

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[28] Burton, p. 231.

[29] Eric Temple Bell, "The Queen of Mathematics," in Vol. 1 of The World of Mathematics, ed. James R. Newman (New York: Simon & Schuster, 1956), p.517.

[30] David Berlinski, A Tour of the Calculus (New York: Vintage Books, 1995), p.59.

[31]Black, p. 770.

[32]Constance Reid, From Zero to Infinity (MAA Spectrum, 1992), p. 15.

Facing a new millennium, the world today takes pains to divorce the spiritual from the temporal. One could argue that modern mathematics has usurped the role that religious doctrine played for so many centuries in the past. Mathematics is truth, truth of the purest form. But as our world evolves and changes, expansion of ideas and thought is inevitable. And resting exactly in the middle of infinity, zero as a number and zero as a concept may yet hold some elements of mystery and discovery.